NAG Toolbox for MATLAB

g01nb

1 Purpose

g01nb computes the moments of ratios of quadratic forms in Normal variables and related statistics.

2 Syntax

```
[lmax, rmom, abserr, ifail] = g01nb(fcase, mean, a, b, c, ela, emu,
sigma, l1, l2, eps, 'n', n)
```

3 Description

Let x have an n-dimensional multivariate Normal distribution with mean μ and variance-covariance matrix Σ. Then for a symmetric matrix A and symmetric positive semi-definite matrix B, g01nb computes a subset, l_1 to l_2, of the first 12 moments of the ratio of quadratic forms

$$R = x^{\mathrm{T}}Ax/x^{\mathrm{T}}Bx.$$

The sth moment (about the origin) is defined as

$$E(R^s), \tag{1}$$

where E denotes the expectation. Alternatively, this function will compute the following expectations:

$$E\left(R^s\left(a^{\mathrm{T}}x\right)\right) \tag{2}$$

and

$$E\left(R^s\left(x^{\mathrm{T}}Cx\right)\right), \tag{3}$$

where a is a vector of length n and C is a n by n symmetric matrix, if they exist. In the case of (2) the moments are zero if $\mu = 0$.

The conditions of theorems 1, 2 and 3 of Magnus 1986 and Magnus 1990 are used to check for the existence of the moments. If all the requested moments do not exist, the computations are carried out for those moments that are requested up to the maximum that exist, l_{MAX}.

This function is based on the function QRMOM written by Magnus and Pesaran 1993 and based on the theory given by Magnus 1986 and Magnus 1990. The computation of the moments requires first the computation of the eigenvectors of the matrix $L^{\mathrm{T}}BL$, where $LL^{\mathrm{T}} = \Sigma$. The matrix $L^{\mathrm{T}}BL$ must be positive semi-definite and not null. Given the eigenvectors of this matrix, a function which has to be integrated over the range zero to infinity can be computed. This integration is performed using d01am.

4 References

Magnus J R 1986 The exact moments of a ratio of quadratic forms in Normal variables *Ann. Économ. Statist.* **4** 95–109

Magnus J R 1990 On certain moments relating to quadratic forms in Normal variables: Further results *Sankhyā, Ser. B* **52** 1–13

Magnus J R and Pesaran B 1993 The evaluation of cumulants and moments of quadratic forms in Normal variables (CUM): Technical description *Comput. Statist.* **8** 39–45

Magnus J R and Pesaran B 1993 The evaluation of moments of quadratic forms and ratios of quadratic forms in Normal variables: Background, motivation and examples *Comput. Statist.* **8** 47–55

5 Parameters

5.1 Compulsory Input Parameters

1: **fcase – string**

Indicates the moments of which function are to be computed.

fcase = 'R' (Ratio)

$E(R^s)$ is computed.

fcase = 'L' (Linear with ratio)

$E\left(R^s\left(a^{\mathrm{T}}x\right)\right)$ is computed.

fcase = 'Q' (Quadratic with ratio)

$E\left(R^s\left(x^{\mathrm{T}}Cx\right)\right)$ is computed.

Constraint: **fcase** = 'R', 'L' or 'Q'.

2: **mean – string**

μ, indicates if the mean is zero.

mean = 'Z'

μ is zero.

mean = 'M'

The value of μ is supplied in **emu**.

Constraint: **mean** = 'Z' or 'M'.

3: **a(lda,n) – double array**

lda, the first dimension of the array, must be at least **n**.

The n by n symmetric matrix A. Only the lower triangle is referenced.

4: **b(ldb,n) – double array**

ldb, the first dimension of the array, must be at least **n**.

The n by n positive semi-definite symmetric matrix B. Only the lower triangle is referenced.

Constraint: the matrix B must be positive semi-definite.

5: **c(ldc,$*$) – double array**

The first dimension, **ldc**, of the array **c** must satisfy

if **fcase** = 'Q', **ldc** $\geq$ **n**;
ldc ≥ 1 otherwise.

The second dimension of the array must be at least **n** if **fcase** = 'Q', and at least 1 otherwise

If **fcase** = 'Q', **c** must contain the n by n symmetric matrix C; only the lower triangle is referenced.

If **fcase** $\neq$ 'Q', **c** is not referenced.

6: **ela($*$) – double array**

Note: the dimension of the array **ela** must be at least **n** if **fcase** = 'L', and at least 1 otherwise.

If **fcase** = 'L', **ela** must contain the vector a of length n, otherwise **a** is not referenced.

7: **emu**$(*)$ **– double array**

Note: the dimension of the array **emu** must be at least **n** if **mean** $=$ 'M', and at least 1 otherwise.

If **mean** $=$ 'M', **emu** must contain the n elements of the vector μ.

If **mean** $=$ 'Z', **emu** is not referenced.

8: **sigma**(**ldsig,n**) **– double array**

ldsig, the first dimension of the array, must be at least **n**.

The n by n variance-covariance matrix Σ. Only the lower triangle is referenced.

Constraint: the matrix Σ must be positive-definite.

9: **l1 – int32 scalar**

The first moment to be computed, l_1.

Constraint: $0 <$ **l1** $\leq$ **l2**.

10: **l2 – int32 scalar**

The last moment to be computed, l_2.

Constraint: **l1** $\leq$ **l2** ≤ 12.

11: **eps – double scalar**

The relative accuracy required for the moments, this value is also used in the checks for the existence of the moments.

If **eps** $= 0.0$, a value of $\sqrt{\epsilon}$ where ϵ is the ***machine precision*** used.

Constraint: **eps** $= 0.0$ or **eps** $\geq$ ***machine precision***.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the arrays **a**, **b**, **sigma**. (An error is raised if these dimensions are not equal.)

n, the dimension of the quadratic form.

Constraint: **n** > 1.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldc, ldsig, wk

5.4 Output Parameters

1: **lmax – int32 scalar**

The highest moment computed, l_{MAX}. This will be l_2 if **ifail** $= 0$ on exit.

2: **rmom**(**l2** $-$ **l1** $+$ **1**) **– double array**

The l_1 to l_{MAX} moments.

3: **abserr – double scalar**

The estimated maximum absolute error in any computed moment.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01nb may return useful information for one or more of the following detected errors or warnings.

ifail $= 1$

On entry, **n** ≤ 1,
or **lda** $<$ **n**,
or **ldb** $<$ **n**,
or **ldsig** $<$ **n**,
or **fcase** $=$ 'Q' and **ldc** $<$ **n**,
or **fcase** $\neq$ 'Q' and **ldc** < 1,
or **l1** < 1,
or **l1** $>$ **l2**,
or **l2** > 12,
or **fcase** $\neq$ 'R', 'L' or 'Q',
or **mean** $\neq$ 'M' or 'Z',
or **eps** $\neq 0.0$ and **eps** $<$ ***machine precision***.

ifail $= 2$

On entry, Σ is not positive-definite,
or **b** is not positive semi-definite or is null.

ifail $= 3$

None of the required moments can be computed.

ifail $= 4$

The matrix $L^{\mathrm{T}}BL$ is not positive semi-definite or is null.

ifail $= 5$

The computation to compute the eigenvalues required in the calculation of moments has failed to converge: this is an unlikely error exit.

ifail $= 6$

Only some of the required moments have been computed, the highest is given by **lmax**.

ifail $= 7$

The required accuracy has not been achieved in the integration. An estimate of the accuracy is returned in **abserr**.

7 Accuracy

The relative accuracy is specified by **eps** and an estimate of the maximum absolute error for all computed moments is returned in **abserr**.

8 Further Comments

None.

9 Example

```
fcase = 'Ratio';
mean = 'Mean';
a = zeros(10, 10);
b = zeros(10, 10);
for i=1:9
  a(i+1, i) = 0.5;
  b(i, i)   = 1;
end
c = [0];
ela = [0];
emu = [0.8;
     0.6400000000000001;
     0.5120000000000001;
     0.4096000000000001;
     0.3276800000000001;
     0.2621440000000001;
     0.2097152000000001;
     0.1677721600000001;
     0.1342177280000001;
     0.1073741824000001];
beta = 0.8;
sigma = zeros(10, 10);
sigma(1,1) = 1;
for i=2:10
  sigma(i,i) = beta*beta*sigma(i-1,i-1)+1;
end
for i=1:10
  for j=i+1:10
    sigma(j,i) = beta*sigma(j-1,i);
  end
end
l1 = int32(1);
l2 = int32(3);
eps = 0;
[lmax, rmom, abserr, ifail] = g01nb(fcase, mean, a, b, c, ela, emu,
sigma, l1, l2, eps)
```

```
lmax =
           3
rmom =
    0.6820
    0.5357
    0.4427
abserr =
   8.4646e-10
ifail =
           0
```